REVISTA CÁTEDRA

La Taptana o contador indígena como estrategia de aprendizaje en operaciones matemáticas básicas

The Taptana or indigenous counter as a learning strategy in basic mathematical operations

Martha Alquinga-Chango
Universidad Central del Ecuador, Quito, Ecuador
mralquinga@uce.edu.ec
https://orcid.org/0000-0001-9746-4079

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Resumen

El aprendizaje de la Matemática se manifiesta como un problema que necesita ser observado, estudiado y abordado desde múltiples perspectivas para lograr entender el fenómeno y actuar sobre él. Desde diversas teorías educativas se plantea la utilización de materiales didácticos para promover un nivel óptimo en la comprensión de los procesos matemáticos y la adquisición de los conceptos fundamentales de esta ciencia.

La Taptana o contador indígena hace referencia al acervo cultural prehispánico de los antiguos habitantes del actual territorio del Ecuador. Su origen posiblemente se remonta a la cultura Cañari, fase Tacalzhapa, 500 a.C., (Uhle, 1922). En los años 80 en el Centro de Investigación para la Educación Indígena (CIEI) se recreó este artefacto de cálculo dando como resultado un material didáctico innovador utilizado especialmente, en lo que hasta hace pocos años atrás se conoció como Sistema de Educación Intercultural Bilingüe.

En el presente estudio se indaga sobre la efectividad de la Taptana, como instrumento matemático para la enseñanza-aprendizaje de las operaciones matemáticas básicas. Se trabajó con estudiantes de dos paralelos, A y B, pertenecientes a Cuarto año de Educación General Básica de la Unidad Educativa José María Velasco Ibarra, institución fiscal situada en el centro norte de la ciudad de Quito. La investigación de corte metodológico cualitativo fue desarrollada en los meses de septiembre, octubre y noviembre de 2019.

Se logró evidenciar la efectividad de la Taptana. Se concluye que la misma mejora la comprensión y el aprendizaje del sistema decimal y de las operaciones matemáticas básicas.

Palabras clave

Contador indígena, enseñanza aprendizaje, operaciones básicas, Taptana.

Abstract

The learning of Mathematics manifests itself as a problem that needs to be observed, studied and approached from multiple perspectives in order to understand the phenomenon and act on it. From diverse educational theories, the use of didactic materials is proposed to promote an optimal level in the understanding of mathematical processes and the acquisition of the fundamental concepts of this science.

The Taptana or indigenous accountant refers to the pre-Hispanic cultural heritage of the ancient inhabitants of the current territory of Ecuador. Its origin possibly goes back to the Cañari culture, Tacalzhapa phase, 500 B.C., (Uhle, 1922, p. 108). In the 80's, the Research Center for Indigenous Education (CIEI) recreated this artifact, resulting in innovative didactic material used especially in what until a few years ago was known as the Intercultural Bilingual Education System.

The present study investigates the effectiveness of the Taptana, as a mathematical instrument for the teaching-learning of basic mathematical operations. We worked with students from two parallel, A and B, belonging to the fourth year of Basic General Education of the José María Velasco Ibarra Educational Unit, a fiscal institution located in the northern center of the city of Quito. The qualitative methodological research was developed in the months of September, October and November 2019.

The effectiveness of the Taptana was demonstrated. It is concluded that it improves the understanding and learning of the decimal system and basic mathematical operations.

Keywords

Indigenous accountant, teaching learning, basic operations, Taptana.

1. Introduction

In Ecuador, as in other places in the world, in numerous occasions the teaching-learning of Mathematics has been associated with certain ideas of complexity, displeasure, clumsiness and distrust in one's own capacity. Other similar thoughts and attitudes probably make this task even more complex than it already is; consequently, it is not strange to observe a kind of social fear, dread and feeling of failure before its study. Different local and international evaluations that measure student performance in Mathematics show discouraging results.

In Latin America the report Third Regional Comparative and Explanatory Study (TERCE) 2013, developed with the participation of 15 countries: Argentina, Brazil, Chile, Colombia, Costa Rica, Ecuador, Guatemala, Honduras, Mexico, Nicaragua, Panama, Paraguay, Peru, Dominican Republic and Uruguay; plus the State of Nuevo Leon (Mexico) shows the academic deficiencies found in the learning of the school population. In TERCE, the complex problem of learning mathematics in the school environment is evident.

In the report issued by the Institute for Educational Evaluation, INEVAL, it is observed that Mathematics continues to have the lowest levels of student performance.

> 70.9% of students in Ecuador do not reach level 2, categorized as basic performance level in mathematics compared to 23.4% of students in member countries of the Organization for Economic Cooperation and Development, OECD... Among low-performing students, 21% are at level 1A, and are only able to perform routine tasks in well-defined situations... (Instituto Nacional de Evaluación Educativa, 2018, p. 44).

Mathematics is a science that is always useful and present at every moment of a person's life. It is the structural tool of the exact sciences and favors the development of logical reasoning. Unfortunately, the situation of mathematics in the classroom is quite complex. The following quote from UNESCO reiterates the dramatic situation of mathematical learning today.

> As for Latin America and the Caribbean, 1 out of every 3 children and adolescents in the region cannot read correctly, as expected for their age, and 1 out of every 2 has serious difficulties in mathematics. Various reports from UNESCO and other international organizations indicate that the minimum levels of competence in mathematics worldwide are low and very low" (United Nations Educational, Scientific and Cultural Organization, UNESCO, 2017).

Thus, in the present work the educational results of the application of Taptana for the teaching of mathematics are analyzed. The aim was to investigate the teaching-learning process of the decimal system and basic operations: addition and subtraction without and with regrouping, in students of the Fourth Year of Basic Education in the "José María Velasco Ibarra" Educational Unit, during the months of September, October and November 2019.

The teaching-learning of Mathematics is a primary research topic. It is of vital importance to look for mechanisms that contribute to the encounter between the student and Mathematics. An effective pedagogical approach is necessary that allows us to live and value mathematics as something beautiful, fun and useful, but not as a problem.

This article consists of three parts. In the first, a brief historical reference about Taptana in its historical-cultural context is presented. In the second part, the methodology used for the application of Taptana and the process of teaching and learning developed using Taptana is presented. The instruments used, as well as the activities and exercises carried out are described. The reasons for the processes developed are also mentioned. In the third part the results achieved are presented. For the data analysis SPSS was used, which is said to be the most complete and used commercial statistical program worldwide (López and Fachelli, 2015). Finally, the conclusions and recommendations of this research are noted.

2. Literature Review

2.1 Brief historical reference of La Taptana

In the province of Cañar and in some sectors of the provinces of Azuay, Chimborazo, Loja, Morona Santiago, El Oro and Guayas, in Ecuador, the Cañari culture was developed. Archaeological remains found in the old Cañari territory, Tacalzhapa phase, 500 B.C. give account of a singular finding, a stone of approximately 38 x 25 centimeters, which has inscribed lateral rows of holes (10) and subdivided lockers, which was given the name of Taptana.

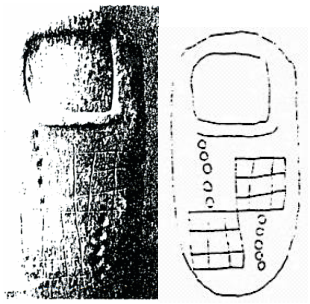


Figure 1. Taptana in stone and drawing of the taptana. Fountain: (Butsch, Calero and Muenala, 1998. p. 6)

The operational processes associated with Taptana probably reflect the thinking and way of life of the Cañaris. It could be affirmed that in this Cañari stone there is evidence of abstract mechanisms that were used to solve daily calculation problems. Grids and circles, according to CIEI researchers, could refer to the management of the decimal system and also to the Andean cosmovision. It could be deduced that it is possible that in the Taptana the ancestral understanding of the world takes shape, in which, data and time can be visualized through the trace of a spiral, unlike the well-known linearity observed in the western logical reasoning.

> ... the grains replace the real elements (animals, agricultural products, etc.) that are the object of the calculation, finding an "indirect" relationship with the referents that are calculated, or, in other words, replacing objective reality with representative reality... (Yánez, 1985, p. 415).

This is how this particular way of operating with the Taptana is described in the Andean indigenous world. It is evident that the assignment of numerical meaning is both arbitrary and systematic. Associating a quantity to maize grains, beans, and beans was useful to calculate quickly and accurately.

The research carried out by CIEI led to the application of Taptana for the teaching-learning of mathematics in the intercultural bilingual education system. In addition, some editorial projects were developed and school texts in Quichua were created. (Valiente, T. and Küper, W., 1998)

The cultural roots of the Andean peoples have great scientific contributions in the fields of agriculture, astronomy and also in mathematics. The Taptana is a cultural contribution from the Andes to the world. The exact use of the Cañari stone called Taptana is not known. It was suggested that it could be a game board. A possible spiral direction of the movement of the grains (when carrying out the mathematical calculations) is associated with the Andean cosmovision. Nevertheless, it is evident that still a greater number of investigations are needed that give account of the cultural and scientific wealth that this artifact lodges.

For decades, Taptana as a didactic resource has been included in several school texts of the Intercultural Bilingual Education System, (understood as the system that brings together the peoples and nationalities of Ecuador). In the book for the subject of Mathematics: "Quimsa Yupaicamayuc Camu" published in 1989, by the author Humberto Muenala mathematical contents are presented that go from counting to multiplication with the use of Taptana.

2.2 Importance of Taptana as a teaching material

The taptana is a sample of the creative, practical and complex mathematical mind of the ancient inhabitants of present-day Ecuador, and specifically of the Cañari people in whose territory vestiges of several taptanas made of stone and wood were found. It will be urgent to socialize this discovery in order to nourish the recognition, respect and value that the scientific production of the ancestral peoples deserves, in short, one of the cultural roots of the Ecuadorians.

The taptana is a kind of abacus that makes mathematical abstractions concrete and makes them manipulable and understandable in a simple and effective way. Assimilating the mechanism of its operation it is possible to elaborate diverse taptanas as structure and materials of manufacture, this way it will be possible to count, in several contexts, with an important educational resource at the time of learning the main mathematical operations.

3. Methodology

The application of the taptana for the teaching-learning of basic mathematical operations was carried out in the José María Velasco Ibarra Educational Unit, with the students of the A and B parallels of the Fourth Year of Basic Education, aged between 8 and 9 years old. Sixty individuals participated.

Wooden boards of approximately 18cm had to be made. X 25cm. (taptanas) on whose surface were engraved three vertical rows of 9 circumferences. The material was complemented with 19 color cards of approximately 1 cm in diameter.

The work was done from a qualitative approach since the educational phenomenon was observed from a humanistic point of view. According to its purpose, this was an applied research. Microsocial, due to the size of the population. It was a primary research since the data was generated by the researcher. Quasi-experimental because no equivalent groups were worked with. And descriptive because it relates the process of teaching and learning basic mathematical operations, in fourth year students of basic education through the taptana.

For the development of the field work we had the collaboration of students from the Faculty of Philosophy, Letters and Education Sciences, of the Pedagogy of Language and Literature Career, the ladies: Andrea Torres Moreno, and Linda Aguilar Jurado; and Mr. Cristian Galarza of the Computer Science Career.

3.1 Applied tests

The diagnostic test that was initially applied consisted of 12 items. Twelve skills were evaluated in relation to the handling of mathematical concepts corresponding to the Fourth Year of General Basic Education and are the following:

1. Recognizes the positional value of a digit in a quantity at the units
2. Recognizes the position value of a digit in a 3-digit amount in the tens position
3. Recognizes the position value of one digit in a 3-digit amount at the hundreds position
4. Reads and writes a three-digit amount
5. Distinguishes a greater quantity from a lesser one
6. Distinguishes a smaller amount from a larger amount
7. Operate the sum without regrouping two summands
8. Operate the subtraction without regrouping two quantities

9. Operate the sum with regrouping to the ten
10. Opera the subtraction with regrouping of the ten
11. Resolves a summation application problem with regrouping
12. Solves a problem of applying subtraction with regrouping

Initially, the proposal was presented to authorities and teachers of the Mathematics subject of the J. M. Velasco Ibarra Educational Unit. The application of the work with the Taptana was carried out in two weekly sessions, of approximately 80 minutes each, during 10 weeks.

The final test was passed once the previous process was concluded. The aim was to evaluate the level of acquisition achieved by the students on the same mathematical concepts and skills evaluated through the diagnostic test.

3.2 Teaching mathematics through Taptana

Mathematical calculation can have different number systems. Historically the systems are known, vigesimal used by the Mayans, quinary, duodecimal, binary, sexagesimal, etc. It can be said that it is possible to organize the calculation from different ideas of grouping quantities. However, given the wide expansion of the decimal system, this is currently the mathematical mechanism used throughout the world.

The decimal system is the one that uses as main reference the number 10 and the position or location of each digit in a certain order. Based on groups of 10 units, each of the orders in the system is created. Thus 10 units form a ten, 10 tens, a hundred, 10 hundreds form a unit of one thousand and so on (Britton and Bello, 1982).

Generally, the handling of this mathematical content, in regular school, is done from the memorization of the different orders mentioned above and it is done through lessons and repetitions. Facing this traditional way of learning mathematics, the physical representation of quantities, their concretion and manipulation was proposed to promote an effective understanding of the decimal system by the students.

Taptana is one of the most versatile didactic materials in terms of sensorially explaining the formation of quantities and their meaning. It manages to show how quantities are formed and how they are operated. Thus, the necessary knowledge to transfer mathematical axioms to new situations and exercises naturally arises. In Taptana, color codes, geometric shapes or objects such as seeds (corn, beans and other grains) and location are used to signify numerical content.

Understanding and acquiring mathematical concepts about the decimal system should be a process that invites reflection, active participation, the enjoyment of conquering knowledge, and experimentation that strips the student of the fear of error. This is a process to which many hours should be dedicated since it is the basis for the development of more complex mathematical concepts.

As mentioned above, Taptana, inspired by the ancient Cañari stone, was designed with reference to the decimal system. For the present investigation we worked with the Taptana that contains three orders: unit, ten and hundred. It is important to indicate that it is possible to design Taptanas with orders up to a million and even more, and even others that express decimal numbers. The logic implicit in the Taptana allows to add ascending or descending orders or positions according to specific work needs. Example:

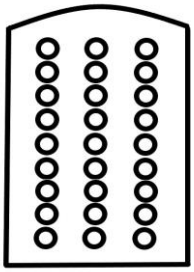
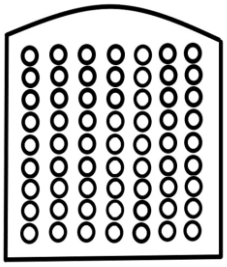
Taptana of 3 orders: unit, ten and hundred	Taptana of 7 orders, from the unit to the million unit
	

Figure 2: Taptana with three orders: units, tens and hundreds /Taptana with 7 orders: unit, ten, hundred, unit of one thousand, ten thousand, hundred thousand and unit of a million. Source: (Elaborated by the author based on general data taken from Butsch, Calero and Muenala, 1998)

It is important to mention that the Taptana in its structure presents the possibility of self-correction since there is one and only one box or place to represent each digit of a quantity and by doing it in a wrong way you can observe the mistake clearly and proceed to correct it.

In the present investigation the teaching-learning of the mathematical concepts with the Taptana was raised through the following 4 stages:

1. Manipulation: place the colored cards on the Taptana to form quantities or to carry out mathematical operations.
2. Graphic representation: drawing and painting using pencil and paper, what was done in the previous stage with the Taptana.
3. Writing the quantity in numbers: write numerically the exercises done.
4. Writing the quantity in words: write in words the numerical results of the exercises performed.

3.2.1 About the process

With due anticipation, the following material was prepared for each pair of students: a wooden board on which was engraved the Taptana, 3 boxes with 19 colored cards: green, blue and red. Green, blue and red paints. Pencils, erasers and pencil sharpeners. Initially it was thought that each student would work individually, however, due to the cramped conditions of the classrooms and the type of furniture, it was decided to work in pairs.

Initially, there was a dialogue with the students to reflect and assume the following work norms:

a) The work tables should be empty to start the task in an organized and comfortable way.
b) The material must remain in order. (the taptana, the tokens and the containers containing the tokens),
c) All students must have a notebook, pencil and paints in the following colors: green, blue and red to perform the exercises.
d) At the end of the representation of each quantity, the cards used must be returned to their container in order to carry out the next exercise.
e) The cards should always be placed from right to left, that is, first the units should be represented, then the tens and so on.

f) The cards should be placed from the base of the cover, never from the top.

The quantities to be worked on the Taptana were selected taking into account the degree of difficulty to go from the simple to the complex, and to go sequentially advancing in the different orders of the decimal system. Through this process, we tried to make the students experience the meaning of the number and try to internalize the abstract mathematical contents.

For each and every one of the exercises, the following process was followed:

Step 1. Once the work to be done was explained, the researcher modeled a first exercise, taking the cards and placing them on the Taptana. When forming the quantities, she did it in order and without hurry. After representing each quantity or performing each exercise, the cards were placed, one by one, in their respective containers. This initial modeling served for the students to work with the Taptana in the same way. It is important to note that the students were very attentive to what they observed, perhaps a little more than to the oral explanation.

Step 2. It was explained that what was done with the taptana would be transferred to the math notebooks. The researcher drew and painted on the blackboard the exercise in question as an example. At this stage, the student was motivated to recall the process, reflect on the experience and refer to it verbally. "... Piaget sees the structures of knowledge built by the subject as a result of his own actions rather than as structures coming from an external source (Coll, 1981, p. 27).

Thus, it is evident the importance of experience as a mechanism that allows the assimilation of learning to the cognitive structures of the individual, as well as that this fact would hardly occur from memory.

Step 3. On the blackboard was written in numbers, the exercise done with the Taptana.

Step 4. On the blackboard, the amount represented on the Taptana was written in words

Once the researcher had finished modeling the process, the students were instructed to develop with their material the same process they had just observed.

3.2.2 Taptana and the teaching-learning of the decimal system

List of proposed exercises:

a)	6, 2, 5, 9, 4, 7, 3, 8, 1				
b)	12	33	57	61	95
c)	146	271	479	617	834
d)	10	20	50	70	90
e)	250	670	490	710	930
f)	209	302	508	705	908
g)	11	222	444	555	888
h)	262	191	828	454	717
i)	123	246	369		

Description of the exercises

a) Single-digit quantities: the association of position and color with the quantity represented was initiated. This process must be done with each and every one of the digits so that students experience the existence of each quantity.

b) Two-digit quantities: the place that each digit occupies in the Taptana was denoted, and in each quantity, that is, so many units and so many tens.
c) Three-digit quantities: the three orders of which this Taptana is composed were used. It is important to point out that you can start this work with a Taptana that contains only two or three orders: units, tens or units, tens and hundreds; and then go on to a Taptana that includes the other orders up to a million.
d) Quantities with zeros: pure tens were represented. The representation of the 0 as the absence, emptiness or lack of a quantity, called the students' attention since somehow the representation of what does not exist was experienced. This fact entails a greater degree of difficulty.
e) Quantities with zeros in the position of the units: The meaning of zero in a quantity was reiterated.
f) Quantities in whose formation an intermediate zero intervenes: it was explained that the zero can be placed in any position in a quantity. Quantities that are fun to represent: numbers that in all the orders had the same amount of chips. This was a fun idea and a discovery for the students, it was highlighted the idea of how pleasant it can be to represent quantities.
g) Symmetric quantities: the figures that form a quantity reinforce the understanding of the number. This idea can be taken up again when working with the other orders of the decimal system. (unit of thousand, ten thousand... million).
h) Quantity in which an inner hierarchy is observed: the exercise was propitious to compare quantities and the way they are formed.

As an example, it is shown the graphic representation of two exercises:

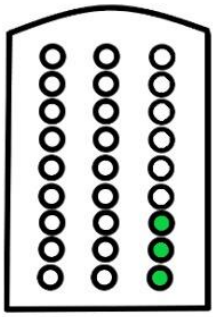
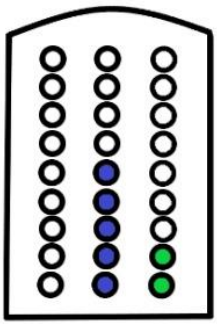
Graphic representation of the number 3	Graphic representation of the number 52
	

Figure 3: Graphical representation of quantities on the Taptana

In each class period of approximately 60 minutes, students followed the instructions in quantity formation and managed to do so in order and successfully. At that time, they were invited to suggest quantities to be represented by all the students in the class. This activity became a challenge and a motivation. It was important to take care not to tire the students and to measure the attention span so that the work is not forced and does not become annoying or boring. Care was taken so that each exercise was done without missing any phase of the process.

It is important to indicate that the individual work rhythm and the particular aesthetics were respected. Each one decided the size and location of the graphic representations of the

exercises performed and other details, at the moment of transferring the exercises to the Mathematics notebook.

3.2.3 Taptana and the teaching-learning process of addition without regrouping

Addition is the mathematical operation in which two or more quantities are put together. Initially and until the students gained confidence and satisfaction in operating summation it was essential to add up amounts without regrouping or "carrying". The regrouping is a complex idea that needs a previous experimentation and exercise.

To operate the sum was followed a process that went from simple to complex. Care was taken with the degree of difficulty that had to be overcome each time and before moving to the next level of complexity.

In each exercise a particular learning process was developed that built a global understanding of the meaning of addition.

List of proposed exercises:

a) 3+4, 5+2, 7+1
b) 11+32 16+31 54+35
c) 223+126 345+234 572+416
d) 351+10 817+120 734+200
e) 406+0 718+0 583+0
f) 703+80 901+60 407+50
g) 1+3+5 4+2+1 6+1+2
h) 31+2 83+5 47+2
i) 234+11+1 451+12+4 352+23+1
j) 10+20+30 40+20+10 20+30+40
k) 200+300+400; 100+600+200; 300+100+400

Description of the exercises:

a) Additions of two quantities, each of them of a single digit.
b) Sums of two quantities, each one composed of two digits. Little by little the students were experiencing the occupation of the different orders at La Taptana. The conceptualization of the meaning of quantity was reinforced.
c) Sums of quantities composed of three digits
d) Sums of quantities including zero
e) Sums of a summation plus zero
f) Sums with zeros in the different orders
g) Additions with three single-digit summands
h) Additions with a two-digit compound summand and a one-digit summand
i) Additions three three-digit summands, two digits and one digit
j) Sums with three exact tens summands
k) Sums of three summands of exact hundreds

As an example, the graphic representation of two exercises is shown:

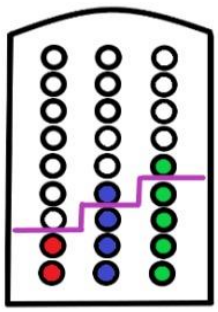
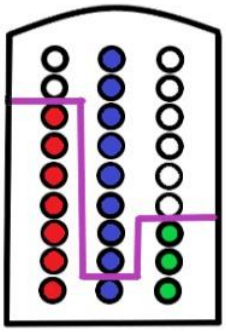
Graphical representation of the sum of 234+11= 245	Graphical representation of the sum of 713+80= 793
	

Figure 4: Representation of the sum without regrouping on the Taptana

3.2.4 Taptana and the teaching-learning process of subtraction without regrouping

Subtraction is the mathematical operation that complements addition and consists of removing or withdrawing one amount from another. We worked on a list of exercises to represent particular aspects of subtraction and thus configure the integral learning of this mathematical operation.

List of proposed exercises:

a) 9-3 7-4 8-5
b) 56-23, 83-41 97-54
c) 489-341 852-741 968-632
d) 618-10 452-30 954-50
e) 60-20 90-30 70-40
f) 601-0 189-0 937-0
g) 893-100 561-200, 957-300

Description of the exercises:

a) Subtractions where the minuendo and the subtrahend consisted of a single digit
b) Subtractions where the minuendo and the subtrahend consist of two digits
c) Subtractions where the minuendo and the subtrahend consist of three digits
d) Subtractions where the minuendo consists of three digits and the subtract is formed by exact tens
e) Remains where the minuendo and the subtrahend were made up of exact tens
f) Subtractions where the subtraction was 0
g) Subtractions where the three-digit minuendo must be subtracted from an exact hundred

As an example, the graphic representation of two exercises is shown:

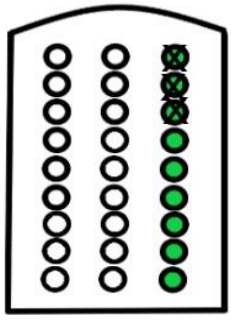
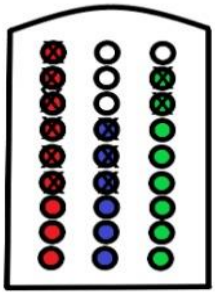
Graphical representation of subtraction 9-3= 6	Graphical representation of the subtraction 968-632= 336
	

Figure 5: process of representation of the subtraction without regrouping in the Taptana

3.2.5 La Taptana and the teaching learning of the sum with regrouping

Once the learning of addition and subtraction without regrouping was overcome, it was possible to advance towards addition with regrouping or "summing carrying". This is the phase of the addition process that usually presents the greatest difficulty due to the level of abstraction demanded. It was important to dedicate the necessary time to provide the required explanations and examples. It was also verified in practice whether the instructions were understood and followed.

Regularly, this process usually consists of verbal repetition of the algorithm learned from memory. Example:

$$\begin{array}{r} 5 \\ +6 \\ \hline 11 \end{array}$$

"... as 5+6= 11, I write 1 and have 1..."

Through an exclusively verbal activity it is difficult to know if the student understands the meaning of the algorithm. It should be indicated that when operating this kind of sums, what is done is to regroup the elements to form a unit of the following superior order. In the case of the 5+6 sum, a group of more than 10 units has been formed. What should be done is to regroup 10 units and take them to the next higher order as a ten. Then write down what resulted, in this case, one unit in the order of tens and one unit in the order of units. As a result of this sum, 11 is obtained.

With the use of Taptana, a cognitive imbalance is visualized. In the Taptana, in the columns of units, tens, hundreds... only 9 spaces exist, for 9 cards and when operating the sum with regrouping 10 or more nonexistent spaces are needed.

The process of addition with regrouping is propitious to explain that in the decimal system groupings of 10 elements in 10 elements are made. Thus, by having 10 or more units in one of the orders, a new group of ten elements must be formed and moved to the next higher order. In the units, when you have 10 green chips you must change those 10 green chips for 1 blue chip that represents 1 ten. When you have 10 blue cards, you must change them for 1 red card that represents 1 hundred; and so on with the following orders of thousands, millions, etc,

The process worked with the Taptana was the following:

1. Place the first sum
2. Place the second summing.
3. Group and "take" to the next order, every time you have 10 or more cards (if you can have up to 19) on the site of the units or tens. worked to the order of hundreds given the curricular content for this level schoolchild).
4. Write the result in numbers and words.

List of proposed exercises:

a) 7+4 8+5 9+3
b) 12+9 45+6 86+7
c) 156+5 478+3 742+9
d) 26+45 64+39 73+ 18
e) 234+358 367+218 652+219
f) 145+371 562+281 421+293
g) 256+166 278+243 589+135
h) 7+3 5+5 6+4
i) 8+12 45+55 67+33
j) 380+190 120+280 670+140

Description of the exercises:

a) Additions between single-digit quantities with regrouping towards tens.
b) Additions between a two-digit amount and a one-digit amount, with regrouping towards the tens.
c) Additions of a three-digit quantity plus a one-digit quantity with regrouping towards the tens.
d) Additions between two-digit quantities maintaining the regrouping of the units towards the tens.
e) Additions between three-digit quantities maintaining the regrouping of the units towards the tens.
f) Additions between three-digit quantities with regrouping from tens to hundreds and without regrouping in units.
g) Additions between three-digit quantities with regrouping in the units towards the tens and in the tens towards the hundreds.
h) Additions between single-digit quantities to complete an exact ten.
i) Additions between tens to complete exact hundreds.
j) Additions with zero in the units and with regrouping of the tens towards the hundreds

As an example, the graphic representation of two exercises is shown:

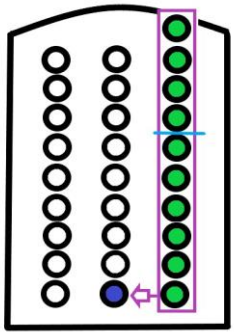
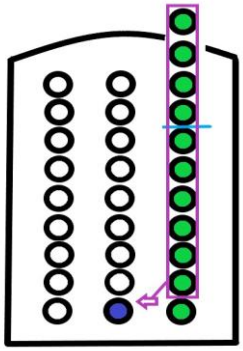
Graphical representation of the sum 6+4=10	Graphical representation of the sum 7+4=11
	

Figure 6: process of representation of the sum with regrouping in the Taptana.

3.2.6 Taptana and the teaching-learning process of subtraction with regrouping

Like addition with regrouping, subtraction with regrouping is a complicated process if the development of understanding is not taken care of. In contrast, the traditional way in which subtraction is usually operated, with the process carried out with the Taptana, the difference can be appreciated.

Generally, it is said:

22

-5

"2 minus 5, I can't afford it, since I can't afford to take 5 out of 2, I borrow 1 and the rest".

In the mind of an 8 or 9-year-old schoolboy this idea is quite incomprehensible due to the level of abstraction required to understand this approach. With Taptana, the student is presented with a process that explains the expression "I can't afford to borrow...

Through the manipulation of the cards on the Taptana, the student can realize the meaning of "borrowing". It becomes evident that one proceeds to disarm or convert, a dozen taken from the immediate superior order, into 10 units. The student can observe the 10 units (green cards) contained in the ten, already loose and appreciate each one, as an individual element. This transformation of the ten into units makes it possible to remove 5 of "2" and continue with the subtraction.

List of proposed exercises:

a) 14-5 23-6 56-8
b) 63-17 71-28 83-59
c) 441-25 6 12-54 863-37
d) 532-143 751-262 854-165
e) 645-150 731-240 846- 160

Description of the exercises:

a) Subtractions where the minuendo has 2 figures, subtracting 1 and regrouping from the ten.

b) Subtractions where the minuendo and the subtrahend consist of 2 figures. The regrouping is done from the ten.
c) Subtractions where the minuendo has 3 figures, the subtracting 2 figures and the regrouping is done from the tens.
d) Subtractions where the minuendo and the subtrahend have three digits. The regrouping is done from the tens.
e) Subtractions where both the minuendo and the subtrahend are quantities of 3 figures and the regrouping is done from the hundred.

As an example, the graphic representation of two exercises is shown:

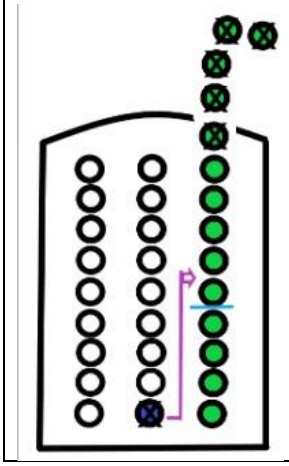
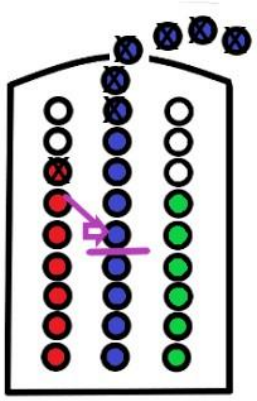
Graphical representation of the subtraction 14-5= 9	Graphical representation of the subtraction 746-60=686
	

Figure 7: process of representation of the subtraction with regrouping in the Taptana

4. Results

At the conclusion of the stage of application of Taptana in the teaching-learning process of basic mathematical operations, the final test was applied to evaluate the results achieved. The statistical program SPSS was used for data analysis. The statisticians for the hypothesis test allow articulating predictions about the results of the research.

The collected results are presented in two parts. In the first one, a hypothesis test is carried out for independent samples. The aim of this application was to know if there were statistical differences between the parallels A and B, in the total scores obtained in the diagnostic and final tests.

The second analysis corresponds to related samples. We sought to verify if there was a statistically significant change between the Diagnostic Evaluation and the Final Evaluation for each one of the items and for the total score.

4.1. Hypothesis test for the difference of two independent populations

In this study, both parallels 4A and 4B are independent populations and the aim is to check whether the scores between the two parallels are the same in the diagnostic and final assessments:

- Null hypothesis (Ho):
  - The score of parallel 4A= The score of parallel 4B
- Alternative hypothesis (H1):

- The score of the parallel 4A≠El score of the parallel 4B
- Test statistic "tobs"
- Ho's rejection criteria:
  - oH is rejected if the bilateral significance<0.05 (in this case a significance level of 5% is used, therefore 0.05 is noted).

CURSE		N	Media	Standard deviation.	Error típ. Of the average
Total diagnostic test score	4to A	27	7,15	2,931	0,564
	4to B	34	6,71	2,154	0,369
Total final test score	4to A	27	11,48	1,122	0,216
	4to B	33	11,21	0,960	0,167

Table 1. Levene test for equality of variances

		Levene test for equality of variances		T-test for equality of means						
									95% Confidence interval for the difference	
		F	Sig.	t	gl	Bilateral significance l	Difference in averages	Error típ. Of the average	Inferior	Superior
Total diagnostic test score	Equal variances have been assumed	2,385	0,128	0,679	59	0,500	0,442	0,651	-0,861	1,745
	Equal variances have not been assumed			,656	46,356	0,515	0,442	0,674	-0,915	1,799
Total final test score	Equal variances have been assumed	0,001	0,981	1,002	58	0,321	0,269	0,269	-0,269	0,808
	Equal variances have not been assumed varianzas iguales			0,986	51,479	0,329	0,269	0,273	-0,279	0,818

Table 2. Levene test for equality of variances

In both cases, equal variances are assumed since in Levene's test the hypothesis of equality of variances is not rejected. Under that assumption, the null hypothesis of equality of scores in the diagnostic test and the final test are not rejected since the bilateral meanings are not less than the 5% set.

4.2 Hypothesis testing for related samples

- Null hypothesis (Ho):
  - The result at diagnosis = The final result
- Alternative hypothesis (H1)):
  - The result in the diagnosis ≠ The final result
- Test statistic "tobs"
- Ho's rejection criteria:

- Ho is rejected if the bilateral significance<0.05 (in this case it was used)

	Related Differences					t	gl	Sig. (bilateral)	Decision
				95% Confidence interval for the difference					
	Media	Deviance típ.	Error típ. de la media	Inferior	Superior				
Recognizes the positional value of a digit in a quantity in the units position	-0,80	0,40	0,05	-0,90	-0,70	-15,36	59	0,000	Rejection Ho
Recognizes the position value of a digit in a 3-digit amount in the tens position	-0,75	0,44	0,06	-0,86	-0,64	-13,30	59	0,000	Rejection Ho
Recognizes the position value of one digit in a 3-digit amount at the hundreds position	-0,80	0,40	0,05	-0,90	-0,70	-15,36	59	0,000	Rejection Ho
Reads and writes a three-digit amount	-0,22	0,42	0,05	-0,32	-0,11	-4,04	59	0,000	Rejection Ho
Distinguishes a larger quantity from a smaller one	-0,10	0,30	0,04	-0,18	-0,02	-2,56	59	0,013	Rejection Ho
Distinguishes a smaller amount from a larger amount	-0,12	0,37	0,05	-0,21	-0,02	-2,43	59	0,018	Rejection Ho
Operates the sum without regrouping two quantities	-0,05	0,22	0,03	-0,11	0,01	-1,76	59	0,083	No Rejection Ho level 95%
Operates the subtraction without regrouping two quantities	-0,18	0,47	0,06	-0,30	-0,06	-3,03	59	0,004	Rejection Ho
Operates the sum with regrouping to the ten	-0,22	0,42	0,05	-0,32	-0,11	-4,04	59	0,000	Rejection Ho
Operates the subtraction with regrouping of the ten	-0,52	0,54	0,07	-0,66	-0,38	-7,46	59	0,000	Rejection Ho
Solves a summation application problem with regrouping	-0,25	0,60	0,08	-0,41	-0,09	-3,23	59	0,002	Rejection Ho
Solves a problem of applying subtraction with regrouping	-0,42	0,62	0,08	-0,58	-0,26	-5,22	59	0,000	Rejection Ho
Total Score	-4,42	2,42	0,31	-5,04	-3,79	-14,16	59	0,000	Rejection Ho

Table 3. Related Difference

From the analysis of the table of related differences, when contrasting the averages of the diagnostic test and the final test by means of the student T-test, one has to

Dexterity 1: Recognizes the positional value of a digit in an amount in the position of the units, statistically significant differences were obtained because the Sig value (bilateral) is less than the p-value 0.05. Therefore, the null hypothesis Ho is discarded and the alternative hypothesis H1 is accepted which indicates that there is significant difference in the development of the mathematical skill: It recognizes the positional value of a digit in a

quantity in the position of the units, after the application of Taptana in the teaching-learning process of the basic mathematical operations: addition and subtraction

Dexterity 2: Recognizes the positional value of one digit in a three-digit amount in the tens position, statistically significant differences were obtained because the Sig value (bilateral) is less than the p-value 0.05. Therefore, the null hypothesis Ho is discarded and the alternative hypothesis H1 is accepted, which indicates that there is a significant difference in the development of the mathematical skill: It recognizes the positional value of a digit in a three-digit number in the tens position, after the application of Taptana in the teaching-learning process of the basic mathematical operations: addition and subtraction.

Dexterity 3: Recognizes the positional value of one digit in a three-digit amount in the hundreds position, statistically significant differences were obtained because the Sig value (bilateral) is less than the p-value 0.05. Therefore, the null hypothesis Ho is discarded and the alternative hypothesis H1 is accepted, which mentions that there is a significant difference in the development of the mathematical skill: It recognizes the positional value of a digit in a quantity of 3 digits in the position of the hundreds, after the application of the Taptana in the teaching-learning process of the basic mathematical operations: addition and subtraction

Dexterity 4: Reads and writes a three-digit amount, statistically significant differences were obtained because the Sig value (bilateral) is less than the p-value 0.05. Therefore, the null hypothesis Ho is discarded and the alternative hypothesis H1 is accepted, which indicates that there is a significant difference in the development of the mathematical skill: Reads and writes a three-digit quantity, after the application of Taptana in the teaching-learning process of the basic mathematical operations: addition and subtraction

Dexterity 5: Distinguishes a greater amount from a lesser one. Statistically significant differences were obtained because the Sig value (bilateral) is less than the p-value 0.05. Therefore, the null hypothesis Ho is discarded and the alternative hypothesis H1 is accepted, which mentions that there is a significant difference in the development of the mathematical skill: Distinguishes a greater quantity from a lesser one, after the application of Taptana in the teaching-learning process of the basic mathematical operations: addition and subtraction

Dexterity 6: Distinguishes a lower amount from a higher one, statistically significant differences were obtained because the Sig value (bilateral) is lower than the p-value 0.05. Therefore, the null hypothesis Ho is discarded and the alternative hypothesis H1 is accepted which indicates that there is significant difference in the development of the mathematical skill: Distinguishes a minor quantity from a major one, after the application of the Taptana in the process of teaching-learning of the basic mathematical operations: addition and subtraction

Dexterity 7: It operates the sum without regrouping of two amounts, no statistically significant differences were obtained because the Sig value (bilateral) is greater than the p-value 0.05. Therefore, the alternative hypothesis H1 is discarded and the null hypothesis Ho is accepted, which indicates that there are no significant differences in the development of the mathematical skill: It operates the addition without regrouping of two quantities, after the application of the Taptana in the process of teaching-learning of the basic mathematical operations: addition and subtraction

Dexterity 8: It operates the subtraction without regrouping two quantities, statistically significant differences were obtained because the Sig value (bilateral) is lower than the p-value 0.05. Therefore, the null hypothesis Ho is discarded and the alternative

hypothesis H1 is accepted which indicates that there is significant difference in the development of the mathematical skill: It operates the subtraction without regrouping of two quantities, after the application of the Taptana in the process of teaching-learning of the basic mathematical operations: addition and subtraction

Dexterity 9: It operates the sum with regrouping to the ten, statistically significant differences were obtained because the value of the Sig (bilateral) is lower than the p-value 0.05. Therefore, the null hypothesis Ho is discarded and the alternative hypothesis H1 is accepted which indicates that there is significant difference in the development of the mathematical skill: It operates the sum with regrouping to the ten, after the application of the Taptana in the process of teaching-learning of the basic mathematical operations: addition and subtraction

Dexterity 10: It operates the subtraction with regrouping of the ten, statistically significant differences were obtained because the value of the Sig (bilateral) is lower than the p-value 0.05. Therefore, the null hypothesis Ho is discarded and the alternative hypothesis H1 is accepted which indicates that there is significant difference in the development of the mathematical skill: Opera the subtraction with regrouping of the ten, after the application of the Taptana in the process of teaching-learning of the basic mathematical operations: addition and subtraction

Dexterity 11: Solves a summation application problem with regrouping, statistically significant differences were obtained because the Sig value (bilateral) is less than the p-value 0.05. Therefore, the null hypothesis Ho is discarded and the alternative hypothesis H1 is accepted which indicates that there is significant difference in the development of the mathematical skill: It solves a problem of addition application with regrouping, after the application of Taptana in the teaching-learning process of the basic mathematical operations: addition and subtraction

Dexterity 12: Solves a problem of application of subtraction with regrouping, statistically significant differences were obtained because the Sig value (bilateral) is lower than the p-value 0.05. Therefore, the null hypothesis Ho is discarded and the alternative hypothesis H1 is accepted which indicates that there is a significant difference in the development of the mathematical skill: It solves a problem of application of subtraction with regrouping, after the application of Taptana in the process of teaching-learning of the basic mathematical operations: addition and subtraction

With the application of the statistical tests described above, it has been technically verified that once the teaching-learning process through Taptana was carried out, a difference was produced that indicates an improvement between the scores reached by the students in the Initial or Diagnostic Test and the Final Test.

Considering the short time in which this experience took place, (10 weeks) it can be said that in general this result would be considered positive. An improvement in the mathematical skills concerning the accomplishment of different exercises on the decimal system and the capacity to operate addition and subtraction without regrouping and with regrouping is evidenced.

5. Conclusions

When analyzing the results of the application of Taptana for the teaching and learning of basic operations: addition and subtraction, without and with regrouping, it was found that Taptana is indeed a didactic resource suitable for this purpose. It was detected a significant

change as far as the skillful handling of the decimal system, and important changes at the level of the capacity to operate sums.

In each exercise carried out, the student experienced the concretion of the number and the mathematical operation; the experience of knowing abstract entities through the senses of touch and sight, possibly strengthened his understanding of mathematics. Rousseau 1985 in the Emilio, referred that children reason very well when they know and have relation with their present, also pointed out that they cannot be asked to reason about what they are unable to understand. The Taptana is a tool that dynamizes the understanding of the implicit logic in the decimal system and in each one of the mathematical operations; its methodology facilitates the transit from the concrete to the abstract, the development of an active and autonomous thought; and consequently, an effective learning of the basic mathematical operations.

Albulquerque, (1953), with respect to the teaching of Mathematics postulated: "... to make everything or knowledge concrete" Thus, to make all knowledge concrete, to link it to the use of the senses would constitute a pedagogical premise to guarantee learning. Since mathematics is a structural science that supports the development of numerous concepts of science, it is essential to continue exploring diverse paths to improve the processes of teaching and learning mathematics. It is recommended to continue with the investigation on the validity of Taptana for the learning of other mathematical concepts such as multiplication, division and square root, given the richness, low cost and versatility of this material. Besides, it is considered indispensable to spread the use of educational resources like Taptana due to its educational, cultural and social contribution.

In Taptana, through activities such as building quantities, operating additions, subtractions, and regrouping, a series of mathematical concepts are given form. According to Radfor, (2006, cited in D'Amore. 2017. p. 125) mathematics is concretized in the planned activity in a time and space and nevertheless, it surpasses the planning, since all the implicit conditions cannot be determined in advance. He states that it is possible to get an idea of what will happen in the classroom, but that this process is not mechanical or deterministic. He emphasizes that the dynamics of work in the classroom will depend on how students and teachers are involved in the activity, on the response to each other, on the active interrelationships in terms of knowledge in general and of the institutions. The conditions for the proposed work with Taptana were favorable and allowed to reach the expected results.

The teaching and learning of basic operations through the use of Taptana is effective. The results obtained demonstrate it. The lack of interest in the search for effective methods to provide mathematical knowledge to children implies a great loss in their integral development. The serious situation of students in the region in relation to basic mathematical skills, shows the importance and urgency of carrying out research that supports the resolution of this problem.

Finally, it is important to urge the development of educational research since it is urgent to propitiate and promote the meeting of the most suitable mechanisms through which the teaching of mathematics can be effectively deployed.

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Author

MARTHA ALQUINGA-CHANGO obtained her Master's degree in Intelligence Development and Education from the Pontificia Universidad Católica del Ecuador Sede Ibarra, PUCE-I, is Coordinator of the Ethnomathematics Area at the Instituto de Investigación en Etnociencias, IIEC-UCE.

She is currently a professor at the Faculty of Philosophy, Literature and Educational Sciences at the Central University of Ecuador. She has taught in the postgraduate courses: Education and Social Development, of the Equinoctial Technological University, UTE and Children's and Youth Literature of the Private Technical University of Loja, UTPL. Her research interest is around logical intellectual functions and creativity. It includes the didactics of Mathematics through ancestral and innovative elements in general, and specifically of the Taptana, a calculation stone or indigenous counter, from the Cañari culture. He has published in the magazine Anales de la UCE.