



Mathematical analysis and simulation in *Matlab* of differential protection in two power transformers Winding

Análisis matemático y simulación en *Matlab* de protección diferencial en dos devanados de transformadores de potencia

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ABSTRACT

This project deals with the realization of mathematical modeling and simulation in Matlab of differential protection in two-winding power transformers, the same one that will be part of the Salesian Polytechnic University, Guayaquil headquarters; will have the objective of simulating the operating conditions of the protection equipment (SEL 587) found in a substation, obtaining the equation that governs the differential protection operations of said relay, by theoretically analyzing the system conditions, both under normal conditions and for any fault event (external fault, internal fault) on the primary and / or secondary side of the transformer, facilitating students who are in the last cycles of the electrical engineering career, learning the concepts and principles of operation of system protections electric power.

PALABRAS CLAVE

Inestabilidad de voltaje, potencias reactivas inductivas, voltajes, ángulos, PV, QV, factores de participación, colapso de red.

RESUMEN

El presente proyecto trata sobre la realización del Modelado Matemático y simulación en MATLAB de la Protección diferencial en transformadores de potencia de dos devanados, el mismo que formara parte de la Universidad Politécnica Salesiana, sede Guayaquil. Tendrá como objetivo simular las condiciones de operación del equipos de protección (SEL 587) encontrado en una subestación, obteniendo la ecuación que gobierna las operaciones de protección diferencial de dicho relé, al analizar teóricamente las condiciones del sistema, tanto en condiciones normales como para cualquier evento de falla (falla externa, falla interna) en el lado del primario o secundario del transformador, facilitando a los estudiantes que cursan los últimos ciclos de la carrera de Ingeniería Eléctrica, el aprendizaje de conceptos y principios de funcionamiento de protecciones de sistema eléctricos de potencia.

INTRODUCTION

Within this document you will find all the information related to the «Mathematical analysis and simulation in Matlab of differential protection in power transformers with two windings», which consists of making all the measurements in a differential protection test module using the 587 relay, in which all the connections and pos-

sible cases of failure that can occur in an electrical power system were made, obtaining real data which allowed analyzing the behavior of the relay for each event.

The first chapter deals with the demand of the problems in the electrical power systems based in daily experience, for which it is proposed to certify the reliability in

Figure 1. Module for transformer protection
Source: Politécnica Salesiana University.



Table 1.
Nomenclature

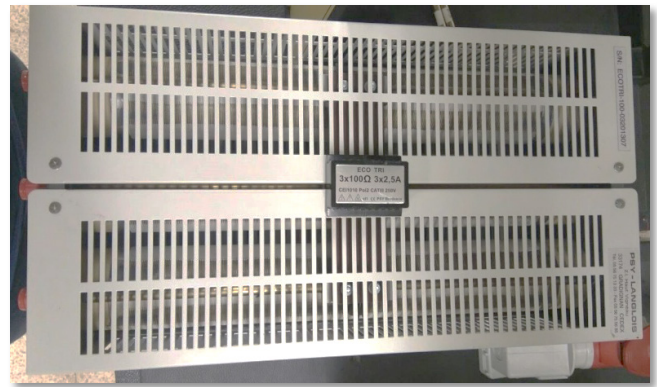
RMS	Effective value
VRMS	RMS voltage
V	Volts
A	Amps
VA	Volt-amperes, unit of apparent power

the distribution networks through differential protection. In the second chapter, the normal operating conditions were reviewed to electrical systems operating for a finite or infinite time under nominal values. In the third chapter, the respective tests were carried out with the different connections that can be made to a power transformer and compared through the program in normal condition and when the internal fault occurs in the transformer. Finally, in the fourth chapter, the mathematical analysis was elaborated through equations which was carried out the structure of the programming in Matlab.

Likewise, the scope of the project and its benefits to society are defined, with the didactic design that allows to develop real tests where the behavior of the relay is analyzed through differential protection, making the different types of connection of the transformers in which they evaluated the internal and external faults in the laboratory that occur normally.

For the understanding and total perception of the subject, books, previous technical projects, papers and web sources were reviewed, in order to consolidate the knowledge regarding the case study.

Figure 2. Variable resistive load from 0-100 Ω , 2,5 A
Sources: Politécnica Salesiana University.



Three-phase Star-Star system

Three-phase Star-Delta system

Three-phase Delta-Delta system

Three-phase Delta-Star system

Ohm, unit of electrical resistance

Effective value

RMS voltage

Volts

Amps

Volt-amperes, unit of apparent power

SCENARIO DESCRIPTION

The experiments were implement using 1 test module, three single-phase transformers each one of 1500 va 120 v / 240 v to form three-phase banks with different connections Star-Star ($\nu\nu$), Star-Delta ($Y\Delta$), Delta-Delta ($\Delta\Delta$), Delta-Star (ΔY), located in the Circuits laboratory of the Politécnica Salesiana University (see Figure 1).

Figure 2 shows a 0-100 Ω variable resistive load three-phase bank, with a maximum current of 2,5 A. The measuring equipment that we use to perform all the tests is the ideal 61-746 (see Figure 2).

This measuring instrument that is displayed in figure 3 was used as a reference to be able to make comparisons of voltages and currents (see Figure 3), allowing you to perform load studies and check the capacity of the electrical systems before adding the load (see Table 1).

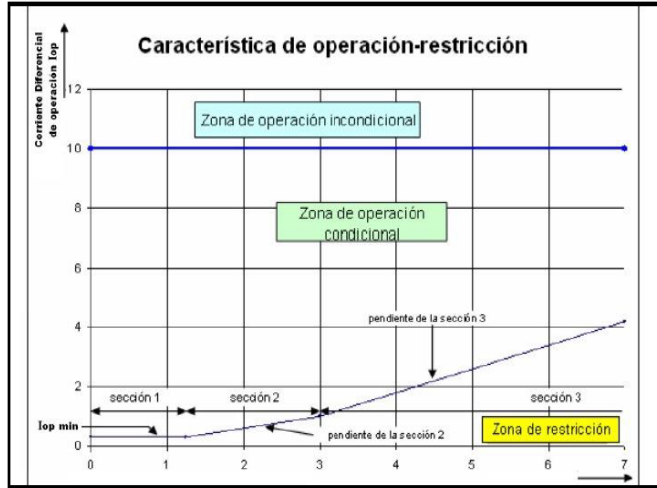
DESCRIPTION OF THE MECHANISM

For a transformer with two winding, the differential relay will detect the faults that occur both inside the protected area and its external connections to the current transformers associated with this protection. This will act as

Figure 3.
Ideal 61-746



Figure 5.
Slope of differential operation



Source: [7, p. 22].

a protection with absolute selectivity; the instantaneous current, modules and phases will be compared.

Figure 4 shows the current flows that circulate through the Tc's which send information to the differential relay, these being governed by the following equations for non-fault and fault-free conditions (see Figure 4):

$$\text{Differential current} = I_d = I_1 + I_2$$

Equation 1: Differential current.

Source: [7, p. 23]

$$I_1 = I_2 \therefore I_d = 0$$

Equation 2: Equipment without failure

Source: [7, p. 23]

$$I_1 \neq I_2 \therefore I_d \neq 0$$

Equation 3: Equipment failed

Source: [7, p. 23].

The differential protection characteristic can be set either as a percentage differential characteristic as a slope or as a variable percentage differential characteristic with double slope (see Figure 5); the element's operation is determined by the operating (I_{OP}) and holding (I_{RT})

Figure 4.
Protection of transformers with two winding

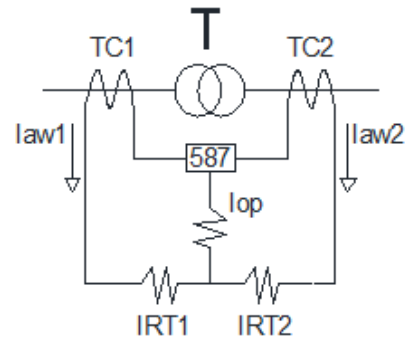
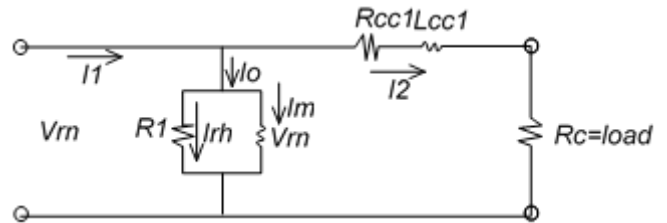


Figure 6.
Equivalent circuit of the transformer



quantities, calculated from the input currents of the windings [7, p. 21].

The figure shows the operating current I_{OP} and a restraining current I_{RT} and an 087P setting or a minimum level required for the I_{OP} operation and two operating slopes called SLP1 with their operating limit I_{RS1} which is an initial curve starting at the origin and with its intersection 087P and a second curve SLP2 which, if used, must be greater than or equal to SLP1 and its entire upper area is a region of operation of the relay and the internal area of the figure shows a region of the relay where this does not operate [7, p. 21].

Triggering occurs if the operation amount is greater than the minimum pickup level and is greater than the curve value, for a particular holding amount. Four settings define the characteristic [7, p. 21].

With careful selection of these settings, the user can closely emulate the characteristics of existing differential current relays [7, p. 21].

Differential protection responds to design criteria based on reliability, speed, selectivity, safety, sensitivity, economy and simplicity [7, p. 21].

II. MATHEMATICAL MODELING

To find the currents of the TC's, the analysis of the transformer is performed, we begin from the equivalent circuit of the transformer where (see Figure 6):

V_{rn} = Input voltage.

R_1 = Hysteresis resistance and heat losses.

L_{m1} = Inductance necessary to produce magnetic flux from the transformer.

R_{cc1} = Short circuit resistance

L_{cc1} = Short circuit inductance

R_c = Load resistance

I_1 = Primary current

I_o = Vacuum current

I_m = Magnetizing current

I_{rh} = hysteresis current

I_2 = Secondary current

Using Kirchhoff's laws, we obtain the following differential equations that define the modeling of the single-phase transformers in figure 6:

Single phase transformer 1

The voltage over time is defined by the following formula:

$$V_{rn}(t) = V_p * \text{Sen}(wt + 0^\circ) \quad (1)$$

From Kirchhoff's law, we define the current of the primary of the transformer T1 as a function of the no-load current and of the secondary:

$$I_{1T1} = I_o + I_2 \quad (2)$$

Knowing that the voltage over time of the inductor is defined as:

$$V(t) = L * \frac{dI(t)}{dt} \quad (3)$$

Applying Ohm's law we draw hysteresis current from the single-phase transformer T1.

$$I_{rh}(t) = \frac{V_{rms}}{R_1} = \frac{V_p * \text{Sen}(wt + 0^\circ)}{R_1} \quad (4)$$

Secondary current of single-phase transformer T1.

$$\frac{dI_2(t)}{dt} = \frac{[V_p * \text{Sen}(wt + 0^\circ)] - [R_{cc1} * I_2(t)] - [R_c * I_2(t)]}{L_{cc1}} \quad (5)$$

Primary current of a single-phase transformer.

$$I_1(t) = \frac{V_{rn}}{R_1} + I_m + I_2 \quad (6)$$

Single phase transformer 2

The voltage over time is defined by the following formula:

$$V_{sn}(t) = V_p * \text{Sen}(wt + 120^\circ) \quad (7)$$

From Kirchhoff's law, we define that the current of the primary of the transformer T2 as a function of the no-load current and of the secondary:

$$I_{1T2} = I_o + I_2 \quad (8)$$

Knowing that the voltage at time of the inductor is defined as:

$$V(t) = L * \frac{dI(t)}{dt} \quad (9)$$

Applying Ohm's law, we draw hysteresis current from the single-phase transformer T2.

$$I_{rh}(t) = \frac{V_{rms}}{R_2} = \frac{V_p * \text{Sen}(wt + 120^\circ)}{R_2} \quad (10)$$

Secondary current of single-phase transformer T2.

$$\frac{dI_2(t)}{dt} = \frac{[V_p * \text{Sen}(wt + 120^\circ)] - [R_{cc2} * I_2(t)] - [R_c * I_2(t)]}{L_{cc2}} \quad (11)$$

Primary current of a single-phase transformer.

$$I_1(t) = \frac{V_{sn}}{R_2} + I_m + I_2 \quad (12)$$

Single phase transformer 3

The voltage over time is defined by the following formula:

$$V_{tn}(t) = V_p * \text{Sen}(wt - 120^\circ) \quad (13)$$

From Kirchhoff's law, we define that the primary current of the transformer T3 as a function of the no-load current and the secondary current:

$$I_{1T3} = I_o + I_2 \quad (14)$$

Knowing that the voltage at time of the inductor is defined as:

$$V(t) = L * \frac{dI(t)}{dt} \quad (15)$$

Applying Ohm's law we draw hysteresis current from the single-phase transformer T3.

$$I_{rh}(t) = \frac{V_{rms}}{R_3} = \frac{V_p * \text{Sen}(wt - 120^\circ)}{R_3} \quad (16)$$

Secondary current of single-phase transformer T3.

$$\frac{dI_2(t)}{dt} = \frac{[V_p * \text{Sen}(wt - 120^\circ)] - [R_{cc3} * I_2(t)] - [R_c * I_2(t)]}{L_{cc3}} \quad (17)$$

Figure 7.

Block diagram of protection relay operation.

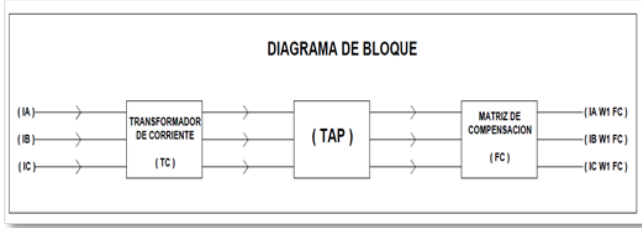


Figure 9.

Parameter entry graph window

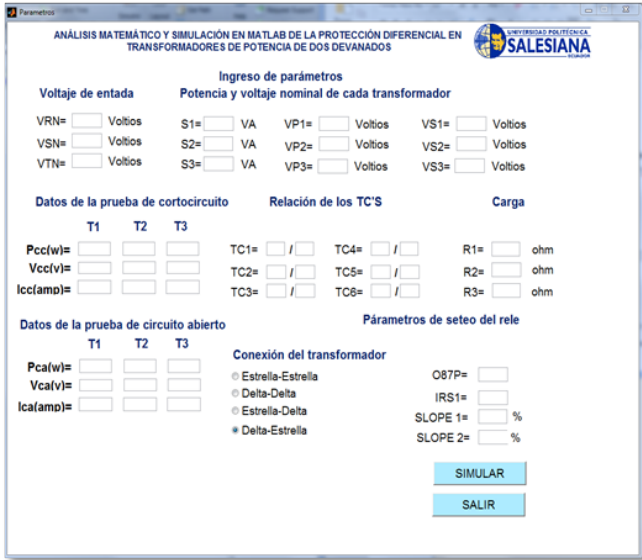


Figure 8.

Compensation matrix

$[CTC(1)] = \frac{1}{\sqrt{3}} \times \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix}$	$[CTC(2)] = \frac{1}{3} \times \begin{bmatrix} 1 & -2 & 1 \\ 1 & 1 & -2 \\ -2 & 1 & 1 \end{bmatrix}$
Matriz para devanado 30° en retraso	Matriz para devanado 60° en retraso
$[CTC(3)] = \frac{1}{\sqrt{3}} \times \begin{bmatrix} 0 & -1 & 1 \\ 1 & 0 & -1 \\ -1 & 1 & 0 \end{bmatrix}$	$[CTC(4)] = \frac{1}{3} \times \begin{bmatrix} -1 & -1 & 2 \\ 2 & -1 & -1 \\ -1 & 2 & -1 \end{bmatrix}$
Matriz para devanado 90° en retraso	Matriz para devanado 120° en retraso
$[CTC(5)] = \frac{1}{\sqrt{3}} \times \begin{bmatrix} -1 & 0 & 1 \\ 1 & -1 & 0 \\ 0 & 1 & -1 \end{bmatrix}$	$[CTC(6)] = \frac{1}{3} \times \begin{bmatrix} -2 & 1 & 1 \\ 1 & -2 & 1 \\ 1 & 1 & -2 \end{bmatrix}$
Matriz para devanado 150° en retraso	Matriz para devanado opuesto a la fase 180°
$[CTC(7)] = \frac{1}{\sqrt{3}} \times \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 1 & 0 & -1 \end{bmatrix}$	$[CTC(8)] = \frac{1}{3} \times \begin{bmatrix} -1 & 2 & -1 \\ -1 & -1 & 2 \\ 2 & -1 & -1 \end{bmatrix}$
Matriz para devanado 150° en adelanto	Matriz para devanado 120° en adelanto
$[CTC(9)] = \frac{1}{\sqrt{3}} \times \begin{bmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix}$	$[CTC(10)] = \frac{1}{3} \times \begin{bmatrix} 1 & 1 & -2 \\ -2 & 1 & 1 \\ 1 & -2 & 1 \end{bmatrix}$
Matriz para devanado 90° en adelanto	Matriz para devanado 60° en adelanto
$[CTC(11)] = \frac{1}{\sqrt{3}} \times \begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$	$[CTC(12)] = \frac{1}{3} \times \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix}$
Matriz para devanado 30° en adelanto	Matriz para devanado de referencia 0°

Primary current of a single-phase transformer.

$$I1(t) = \frac{Vtn}{R3} + Im + I2 \quad (18)$$

$$IBw2 = \frac{I1(t)}{Tc2} \quad (22)$$

Single phase transformer 1

Primary current of transformer T1, seen from the secondary of TC's.

$$IAw1 = \frac{I1(t)}{Tc1} \quad (19)$$

Single phase transformer 3

Primary current of the transformer T1, seen from the secondary of TC's.

$$ICw1 = \frac{I1(t)}{Tc1} \quad (23)$$

Secondary current of transformer T1, seen from the secondary of TC's.

$$IAw2 = \frac{I1(t)}{Tc2} \quad (20)$$

Secondary current of transformer T1, seen from the secondary of TC's (see Figure 7).

$$ICw2 = \frac{I1(t)}{Tc2} \quad (23)$$

Single phase transformer 2

Primary current of transformer T1, seen from the secondary of TC's.

$$IBw1 = \frac{I1(t)}{Tc1} \quad (21)$$

TAP 1 of a differential relay.

$$TAP 1 = \frac{MVA \times 1000 \times C1}{\sqrt{3} \times Vp \times CTR1} \quad (24)$$

TAP 2 of a differential relay.

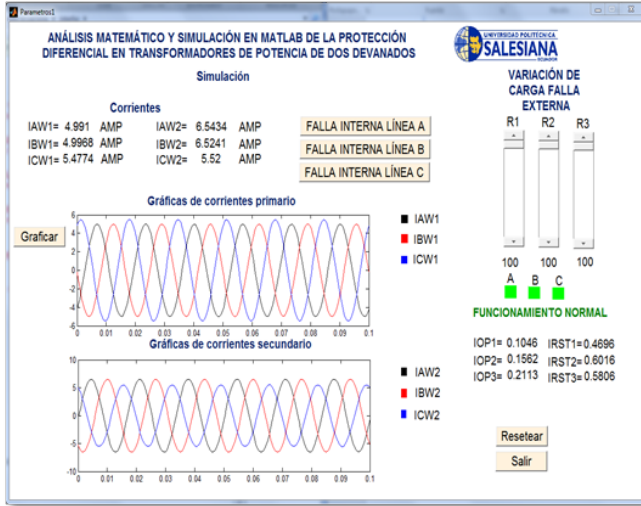
$$TAP 2 = \frac{MVA \times 1000 \times C2}{\sqrt{3} \times Vp \times CTR2} \quad (25)$$

Secondary current of transformer T1, seen from the secondary of TC's.

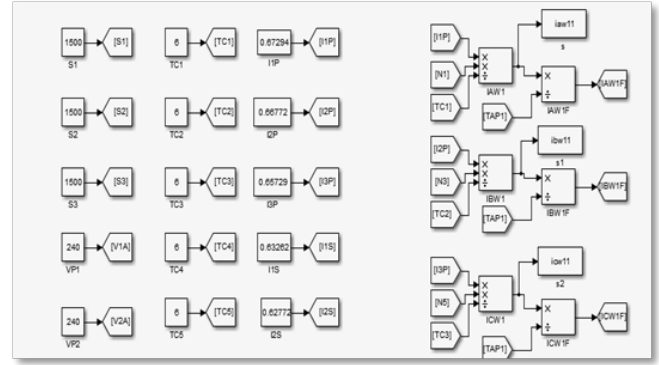
After finding the currents of said TAP which we define with the following formulas:

Figure 10.

Results graph window


Figure 11.

Simulink blocks


TRANSFORMER 1

Primary phase A current, after passing through TAP 1.

$$IAw1F = \frac{IAw1}{Tap1} \quad (26)$$

Secondary phase A current, after passing through TAP 2.

$$IAw2F = \frac{IAw2}{Tap2} \quad (27)$$

TRANSFORMER 2

Primary phase B current, after passing through TAP 1.

$$IBw1F = \frac{IAw1}{Tap1} \quad (28)$$

Secondary phase B current, after passing through TAP 2.

$$IBw2F = \frac{IAw2}{Tap2} \quad (29)$$

TRANSFORMER 3

Current of phase c of the primary, after passing through TAP 1.

$$ICw1F = \frac{ICw1}{Tap1} \quad (30)$$

Secondary phase c current, after passing through TAP 2.

$$ICw2F = \frac{ICw2}{Tap2} \quad (31)$$

Then go to the block of compensation matrices depen-

ding on the transformer connections and their phase difference that was chosen internally in the program and in turn the differential relay relates them through previously adjusted parameters, the matrices are as follows (see Figure 8).

Example:

Protection relay operation through compensation matrix.

Primary current in each of the phases, from the compensation matrix.

$$\begin{bmatrix} IAw1Fc \\ IBw1Fc \\ ICw1Fc \end{bmatrix} = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} IAw1F \\ IBw1F \\ ICw1F \end{bmatrix} \quad (32)$$

Secondary current in each of the phases, from the compensation matrix.

$$\begin{bmatrix} IAw2Fc \\ IBw2Fc \\ ICw2Fc \end{bmatrix} = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} IAw2F \\ IBw2F \\ ICw2F \end{bmatrix} \quad (33)$$

After having had all these currents, the relay proceeds to calculate the operating currents (Iop) and restriction (Irst) for which the following equation is used:

Operating current in phase A.

$$Iop_A = I_A W_1 F C_1 + I_A W_2 F C_1 \quad (34)$$

Operating current in phase B.

$$Iop_B = I_B W_1 F C_1 + I_B W_2 F C_1 \quad (35)$$

Operating current in phase c.

Figure 12.
Internal fault Line A.

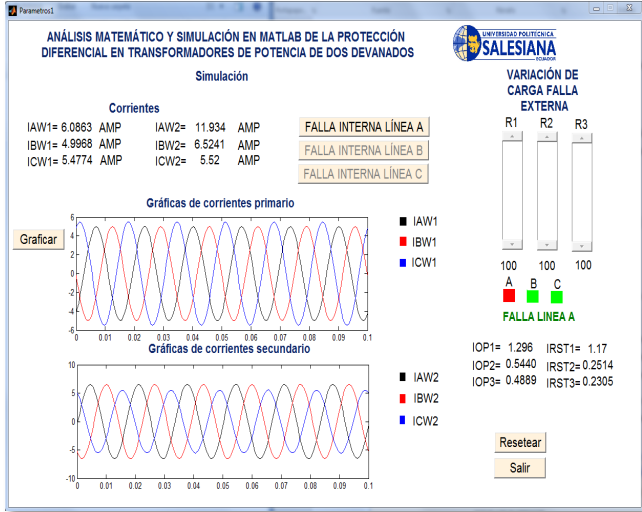


Figure 14.
Primary and secondary current in each phase seen from the CT's, with 100% load

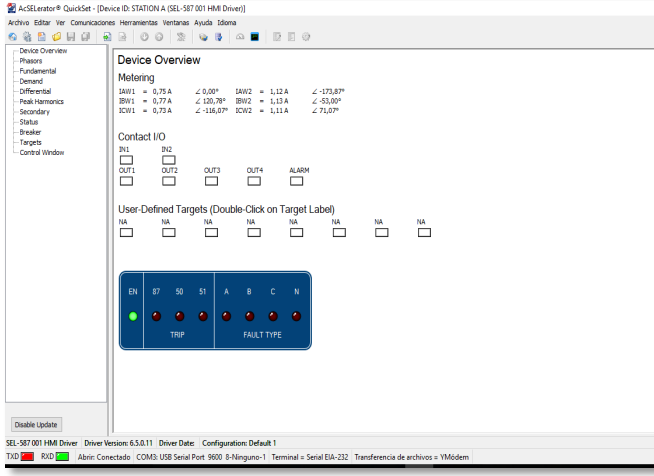


Figure 13.
Normal operation

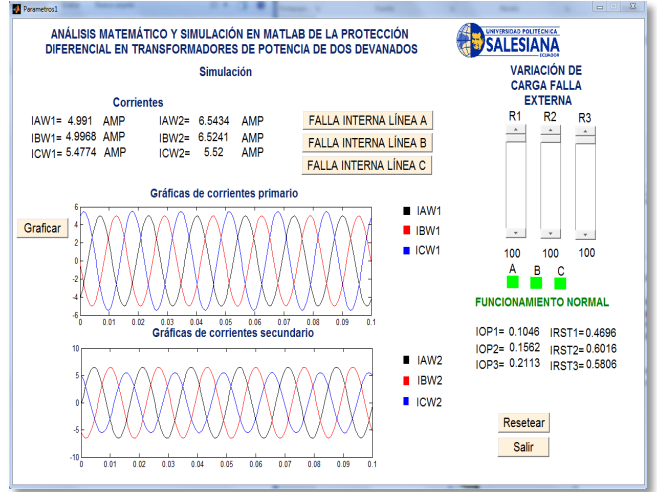
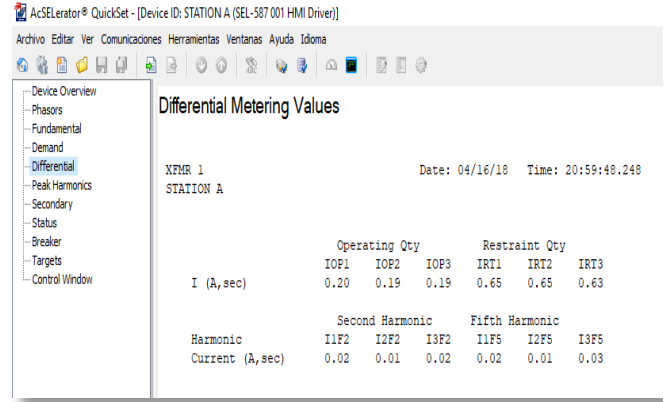


Figure 15.
Operating and restriction currents in each of the phases, with 100% load



III. TEST AND VALIDATION

After obtaining the electrical parameters of each of the transformers through short-circuit and open-circuit tests, the different connections are made under vacuum and under load. The responses of the software whose interface were compared with the graphs obtained by the measurement instrument, resulting in the following (see Figure 9):

The graphs of each of the phase currents that are seen by the relay on both the secondary side and the primary side are shown (see Figure 10).

For the coding of the different graphs the following process was used, at the moment of executing the Simulink internally, figure 11 block arrangements were created that have the equations that represent our modeled system and to be able to obtain the current graphs (see Figure 11):

$$Iop_C = I_C W_1 FC_1 + I_C W_2 FC_1 \quad (36)$$

Restriction current in phase A.

$$Irst = \frac{[I_A W_1 FC_1 + I_A W_2 FC_1]}{2} \quad (37)$$

Restriction current in phase B.

$$Irst = \frac{[I_B W_1 FC_1 + I_B W_2 FC_1]}{2} \quad (38)$$

Restriction current in phase C.

$$Irst = \frac{[I_C W_1 FC_1 + I_C W_2 FC_1]}{2} \quad (39)$$

Operating conditions when the relay operates.
 $Iop \approx 0$ relay not actuated, normal operation.
 $Iop \neq 0$ relay actuated, fault operation.

INTERNAL FAILURE IN EACH OF THE PHASES

By activating any of this «push button» will allow us to see the behavior of the relay in each of the phases as shown in figure 12 (see Figure 12).

Here you can see the current values in the phase where the fault occurred, and also see the operating current in the phase where it occurred.

The phase where the failure occurred will be shown in red and in turn it will proceed to block any load change that could be made in the Slider as well as the «push button» of the failures in the other phases, all this will be blocked until do not press the reset button which will return the readings to normal and clear the fault as shown in figure 13 (see Figure 13).

After reviewing the required results, we press the exit button which will ask us for an exit confirmation and by pressing «yes», the interface will be completely exited and the previously entered data will be deleted (see Figures 14 y 15 and Table 2).

TESTS AND RESULTS

Star-star connection:

Test 1:

R1 = 100,6 ohms

R2 = 100 ohms

R3 = 99,8 ohms

IV. CONCLUSION

The objective of this work was to show by means of the simulation in Matlab the behavior of the differential protection using the SEL 587 relay, of the «Module for transformer protection» which was analyzed to obtain the governing equations, comparing values and the operation of the relay both empty and loaded, the following was concluded:

A didactic modeling was carried out in Matlab where different practices and simulations were performed, in order to visualize and analyze the moment when the differential protection acted for both internal and external failure of a transformer, with the data obtained through the simulations it can be conclude that the operating results of the relay in the simulator are within the range of the trip, since it works at close operating currents of the real relay.

When comparing the responses obtained when simulating the system with the different types of connections of the single-phase transformers, the data obtained were satisfactory, since it was possible to appreciate the currents of the windings both on the primary side and on the secondary side. It was able to appreciate the operating current and restriction of the relay, which showed an error rate of less than 5%.

For the different practices, the parameters were modified in the SEL 587 relay as: (connection type of

single-phase transformers and operating current I_{op}). Through these practices it can be concluded that the load varies in percentage form for the different types of connection in the transformers.

The purpose of the SEL 587 differential relay is to protect the power transformer where the input current must be equal to or similar to the output current, in which real faults that normally occur in electrical power systems could be simulated. reliability in the system and no damage occurs.

The tests were performed on the board «Transformer protection module» which was demonstrated and analyzed the fault that eventually occurs in two-winding transformers, where the differential relay is in charge of protecting the transformer from an internal fault for this, comparison tables were made to consider the settings in the relay and to be reliably in the system.

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